Deployment strategies of mobile sensors for monitoring of mobile sources: method and prototype

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Abstract: *In this communication, the characterization of mobile sources is studied. A quasi online identification method based on the iterative regularization of ill-posed inverse problem is proposed, it allows monitoring of evolutionary phenomena. Several strategies of choice of sensors (Adaptive selection of relevant sensors within the fixed network or management of a pack of mobile sensors) are discussed. To illustrate the approaches, an experimental prototype has been designed in the context of thermal engineering. The experiment is described: design, characterization, exchanges protocols of signals acquisition, movement of mobile elements (sources and sensors).*

Keywords: inverse problem, parametric identification, thermal procedure, Conjugate Gradient Method, Experimentation

I. INTRODUCTION

The distributed parameters identification (continuously depending on space and time) in systems described by nonlinear partial differential equations (PDE) is, in general, difficult. A typical approach is to minimise an output error problem. For example, to minimize a quadratic criterion describing the difference between outputs predicted by the mathematical model and observed outputs collected by using sensors. This approach can be defined as an inverse problem [1].

In more specific context of thermal engineering, inverse problems of heat conduction studied to identify properties, heat flows or initial conditions, are usually ill-posed [2]. In fact, the stability condition (in Hadamard's sense [1]) is generally not satisfied because of weak disturbances on measurements generate large variations on the identified parameters. This can be shown in a matrix environment where, in the numerical solution of partial differential equations system, assembled matrices obtained by finite element method [3] are ill-conditioned.

Matrices inversion is not relevant in the presence of uncertainties on measurements [4-5]. In this context, Tikhonov has developed methods allowing the construction of stable solutions [1-2, 6]. These methods are generally called as "regularization method ". The main principle is to formulate a new problem considering an additional parameter (the regularization parameter) such that the new problem is stable. The construction of these new problems is not trivial. Among various existing approaches, the Conjugate gradient method [7] allowing the iterative minimization of the output error has a regularisation property in context of the inverse problems of heat conduction.

Indeed, this method leads to solve iteratively three problems well posed [8]. In [2], Alifanov says: "*such a method of damping the instability when specifying an approximate solution for an ill-posed problem is based on viscous properties of numerical algorithms of optimization*". This stabilizing effect is illustrated in [9] for an academic situation corresponding to a 1D geometry for which analytical solutions are available. It is highlighted that the main structure of the unknown parameter is identified from early iterations.

The Conjugate gradient algorithm behaves as a filter able to reject disturbances on measurements during the iterative process of minimization. The number of iteration is often considered to be the parameter of regularization of these methods.

This inverse problem resolution is traditionally implemented offline: identification begins when all data are collected [10-13]. However recent adaptations have shown its potential for a quasi-online use in a numerical context [14]. To illustrate the interest of this new approach in a real situation, experimentation was designed for the identification of heat mobile sources (See Fig. 1). This main objective of the prototype is to estimate the trajectory and the heating of one or more surfacic heating sources from temperature measuring. For quasi online identification, the problem of optimum selection of sensors can be posed. Two strategies are discussed in this paper: the first concerns the selection quasi-online of most relevant localized sensors within a network of sensors, the second works on the problem of the intelligent movement of some mobile sensors.

In this communication, the complete description of the experimental device is presented. The choice of dimensions and materials are justified. Heat sources are detailed and an identification procedure of heat flux density is proposed. The acquisition of collected data is presented. Heat elements and sensors are embedded on mobile robots (Khepera III) and the procedure is detailed. The methodology for the identification is presented and the innovative aspects (procedure quasi-online, strategies of sensors choice) are explained. Finally, conclusions and outlooks are briefly trained.

II. EXPERIMENTAL DEVICE

In this part, the studied thermal situation is described. It is intended to illustrate the quasi online resolution of an inverse problem of heat conduction using the iterative adjustment of the conjugate gradient method. It also allows testing strategies of sensors choice (positioning, deployment).

A. Field of study

In this study, the hypothesis of a geometry 2D (infinitely fine plate) was considered in order to reduce the computing times. This assumption is valid only if we can neglect the heat transfer in the thickness of the plate. To do this, it is necessary to consider a fine plate of a metal having a high thermal conductivity. For financial reasons, aluminum was chosen (gold, silver and copper not being possible). This square plate 3m of the each side posed horizontally on a support providing insulation. This last is composed of Rockwool thermal insulation to high temperatures.

In order to verify that the heat transfers are surfacing on the aluminum plate, two mathematical models were compared. The first system (1) corresponds to the selected assumption: the temperature $\theta(x,y;t)$ in each point (x,y) of the plate Ω and at every moment $t \in T$ satisfied:

$$\begin{cases} \forall (x,y;t) \in \Omega \times T & \rho c \dfrac{\partial \theta}{\partial t} - \lambda \Delta \theta = \Phi \\ \forall (x,y) \in \Omega & \theta(x,y;0) = \theta_0 \\ \forall (x,y;t) \in \partial\Omega \times T & -\lambda \dfrac{\partial \theta(x,y;t)}{\partial \vec{n}} = 0 \end{cases} \qquad (1)$$

where ρc is the volumetric heat capacity in $\mathrm{J.m^{-3}.K^{-1}}$, λ is the thermal conductivity in $\mathrm{W.m^{-1}.K^{-1}}$, θ_0 in K is the room temperature (equal to that of the surrounding medium), $\vec{n}$ is the normal unit vector at the border $\partial\Omega$.

The flux $\Phi(x,y;t)$ corresponds to the surface heat sources ϕ_{heat} and also considers a convective exchange with the upper part of the plate as well as a perfect isolation on the underside of the plate.

$$\Phi(x,y;t) = \frac{\phi_{heat}(x,y;t) - h\left(\theta(x,y;t) - \theta_0\right)}{e}$$

where h is the convective heat transfer coefficient in $\mathrm{W.m^{-2}.K^{-1}}$ and e is the thickness of the plate in m.

The second system (2) corresponds to the aluminum plate of $9\ \mathrm{m}^2$, the thickness e posed on the Rockwool plate with the thickness $e_r = 4.5$ cm. This is Rockwool panel, mono-density, semi-rigid, covered with a vapor barrier kraft polyethylene. The reference of this material is Rockwool-Rockmur-Kraft.

The mathematical model describes the heat transfer in 3D geometry is the following:

$$\begin{cases} \forall(x,y,z;t)\in\Omega\times T & \rho c\dfrac{\partial\theta}{\partial t}-\lambda\Delta\theta=0 \\ \forall(x,y,z)\in\Omega & \theta(x,y;0)=\theta_0 \\ \forall(x,y,z;t)\in\partial\Omega_{lat}\times T & -\lambda\dfrac{\partial\theta}{\partial\vec{n}}=0 \\ \forall(x,y,z;t)\in\partial\Omega_{sup}\times T & -\lambda\dfrac{\partial\theta}{\partial\vec{n}}=-e\Phi \\ \forall(x,y,z;t)\in\partial\Omega_{inf}\times T & -\lambda\dfrac{\partial\theta_p}{\partial\vec{n}}=\dfrac{1}{R}(\theta-\theta_0) \end{cases} \quad (2)$$

where $\partial\Omega_{lat}$ is the lateral surface of the aluminum plate and $\partial\Omega_{sup}$ (resp. $\partial\Omega_{inf}$) the superior face (resp. inferior). The coefficient R represent thermal resistance of surface, also called thermal isolation coefficient of surface and expresses itself in $m^2.K.W^{-1}$. In order to consider whether the assumption of heat transfer two-dimensional is valid, the predicted temperatures by the models (1) and (2) are compared for a set of realistic parameters indicated in table I.

TABLE I

INPUT PARAMETERS OF MODELS (1) AND (2) IN UNITE WHERE

$\rho c = 2.4 \quad 10^6$	$\lambda = 237$	$h = 15$	$R = 1.2$
$\phi_{heat}(x,y;t) = 10^5\, e^{-10^3 d^2} \sin\left(\dfrac{2\pi t}{1200}\right)$			$\theta_0 = 291$

In table I, $d(x,y)$ is the distance between the point (x,y) and the center of the superior face of the aluminum plate. It may be noted that the configuration 3D of this study is axisymmetric. The systems of partial differential equations (1) and (2) are solved by using the finite element method [3] implemented by the Comsol code interfaced with Matlab. On figure 1, the evolutions of temperature in various points of the superior face of the plate are submitted to a plate thickness $e = 2$mm.

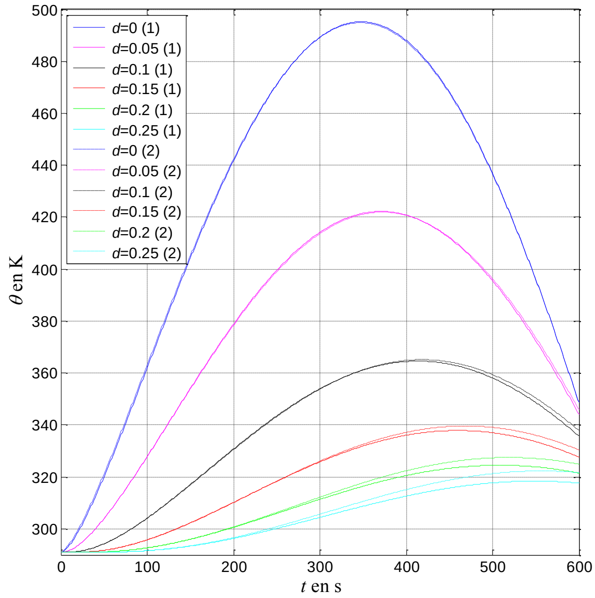


Fig. 1. Comparison of the 2D and 3D models

Figure 1 shows that the 2D model is very satisfactory for points near the heating source. For points far away from the source, weak errors of model will be present. The following table indicates in various points the mean absolute error between two models, as well as the absolute average deviation of temperature *emod* between 2 points located on both sides of the plate (information *ecart* obtained with the model (2)).

TABLE II

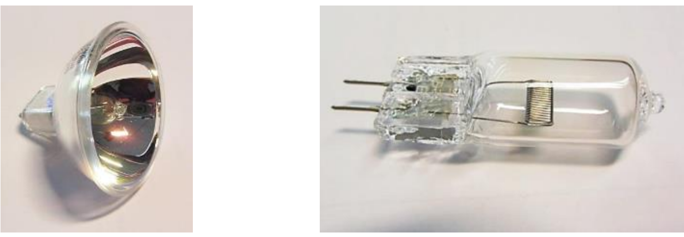
COMPARISONS BETWEEN TWO MODELS

d	0 cm	5 cm	10 cm	15 cm	20 cm
emod	0.5 K	0.5 K	0.5 K	0.8 K	1.2 K
ecart	0.3 K	0.02 K	0.003 K	0.002 K	0.001 K

Considering figure 1 and table II, the assumption of the two-dimensional transfers is retained for an aluminum plate of 2mm thickness posed on panels of Rockwool thickness 4.5cm. The mass of such a plate of 9 m^2 is slightly lower than 50 kg.

B. The heat sources

Heating sources without contact was chosen in order to avoid the inherent problems in movements of the mobile sources. They are halogen bulbs Philips of two different types (24V, 250W, GX5.3) and (36V, 400W, GY6.35), driven by programmable power supplies. These bulbs allow a radiative heating $\phi_{heat}(x,y;t)$ which we suppose that the temporal component is only related to controlling the power supply. Indeed, the delay inherent in the heating of the filament is considered negligible compared with the duration of the experimentation (approximately 30 minutes) and with dynamic delays for the rise in temperature. Thus, it is justified to write for each source $\phi_{heat}(x,y;t)=\phi(t)\,f(x,y)$ where $\phi(t)$ is function of the power provided by the power supply and $f(x,y)$ is spatial distribution of the heat flux density on the plate. Identification campaigns must be realized to know these parameters. The function $f(x,y)$ depends on the geometry of the bulb and the distance to the plate. A reflector placed behind the bulb allows increasing the flux received by the plate. This last depends on the distance between the heat source and the plate. If a constant flux on a delimited surface is desired then a kaleidoscope (device for homogenizing the flux) can be used; see an application in [15]. This last will require that the distance between the plate and the heat source is fixed (equal to the length of the kaleidoscope). In a general case, it is necessary to identify the spatial distribution $f(x,y)$. It is easy (infrared thermography) to ensure that heating is axisymmetric and it is thus to identify $f(d)$ where $d(x,y)$ is the distance between the point (x,y) and the center of the heating source. Without losses of the generalities, we can suppose that $f(d)$ can be written as continuous piecewise linear function and proceed a simultaneous identification of $f(d)$ and ϕ (for several powers) using calibrated experimentations associated with the conjugate gradient method [4]. This must be done for various distances between the source and the plate.



24V, 250W, GX5.3 | 36V, 400W, GY6.35

Fig. 2. The heating sources

C. Sensors

For the temperature measurements, pyrometers were chosen in order to realize an acquisition without contact (which avoid the disturbing medium and facilitate the displacement of the sensors). These are Optris® CSlaser-LT-CF1 delivering an output current in the range 4-20mA. The temperature range is [273,773] in K. Each pyrometer is a cylinder approximately 10 cm long and 5 cm in diameter for a mass of 600g. The resolution of temperature is of 0.1K, the accuracy of 1% and the response time (at 90%) is 150ms. The measurement distance between the pyrometer and the plate is 7cm and the diameter of the measuring disk (corresponding to 90% of the radiation emitted) is 1.4mm. Therefore, the pyrometers provide an average temperature on zones from approximately $2\,\mathrm{mm}^2$.

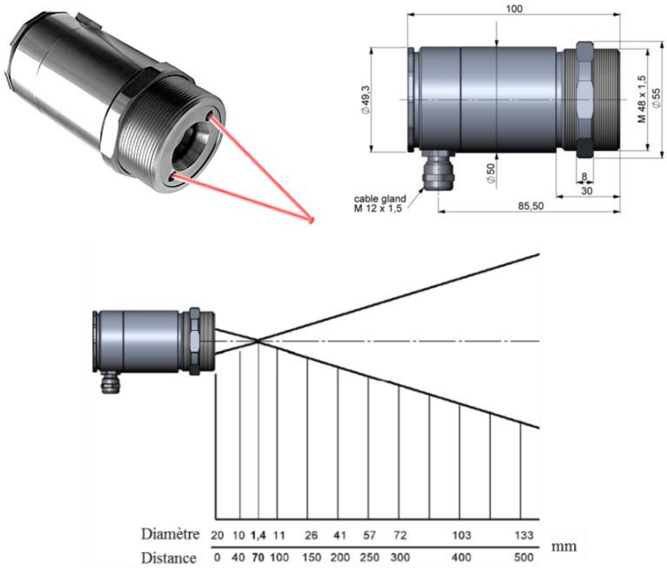


Fig. 3. Pyrometer Optris® CSlaser-LT-CF1

D. Robots

In order to move the sources and the sensors, Khepera-III robots have been selected. These are designed by the Swiss Federal Institute of Technology in Lausanne and developed by the K-TEAM company. The diameter of each robot is 13cm and its height is 7cm. They are able to move a mass lower than 2kg at a maximum speed of 50cm/s. The autonomy is 8 hours (at constant speed and without the embedded platform

Korebot-II) while the duration of the experimental campaigns is envisaged about 30 minutes.

The robots are connected to the management computer via a wireless router (wifi router) through which they can exchange information in the form of frames using the TCP/IP protocol (client/server).

In order to obtain the precise position of each robot, a visible camera (camera AVT StingrayFireWire, F-046C, field of view 61.9° – 76.7°) is wired with the management computer. This camera has 780x580 pixels and a frequency acquisition at 55Hz. By using the software SSL-vision [16-17], the displacement of the robots on the plate is registered and the coordinates as well as the orientations of the robots are sent to the management computer to recalibrate. In fact, the robots have various sensors (10 infrared sensors, 5 ultrasonic telemeters) and wheels with incremental encoders which allow knowing their relative positions but these would suffer an important drift. By using the camera, the accuracy of the position of the robots is about the centimeter.

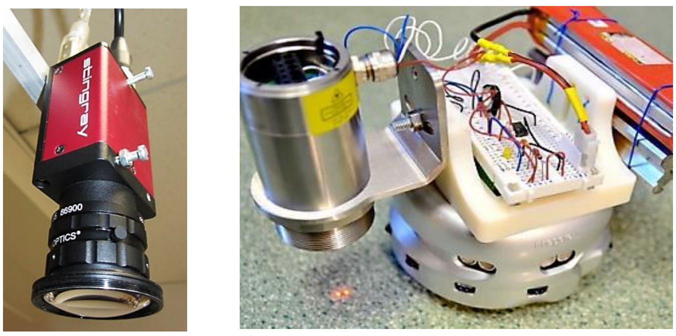


Fig. 4. Camera and Khepera-III robot equipped with a pyrometer

For each robot equipped with a pyrometer, the temperature is measured and the analogical values (currents) are processed by a digital analogical converter (integrated in I/O card KoreIO connected on the expansion card). The robot then sends a frame to the management computer (by using the protocol TCP/IP client/server) containing the position and the temperature recorded at that position and the time and date of acquisition of the measure.

E. In practice…

The temperatures as well as the precise positions of the robots are measured every second. The robots embedding the heating sources can move continuously. The robots embedding the pyrometers move according to the provided indications every second by the management computer (possibly connected to a second computer dedicated to the resolution of the problem of identification).

F. Vision system prototype

The heating sources are embedded on mobile robots which deploy on a plate. The power supply of heat sources is controlled so as to describe various profiles of evolution. On this same plate, several mobile robots are equipped pyrometers in order to measure the temperature of the plate in a quasi-punctiform manner. The measured temperatures and the position of the measurement points are transmitted to a central calculator via a wireless technology (WiFi). The inverse problems for identification will be then solved quasi online. The trajectories of the pyrometers are estimated throughout the process and sent to observant robots which move then to their next positions [15]. The absolute position of the robots being necessary to the system performance (robustness of the identification procedure), a vision system of global localization is used (in complement of odometric measurement). A camera is positioned above the plate; the various measures of location obtained by vision are synchronized with those sent by the observant robots for a possible correction. The device presented in this paper is shown schematically in figure 5.

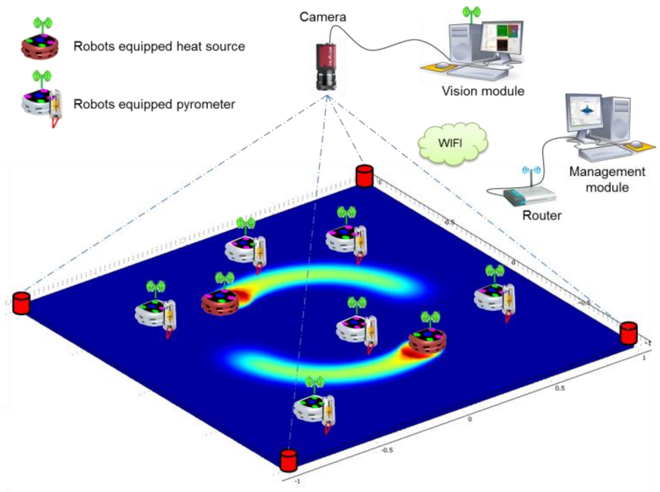


Fig. 5. Vision system prototype

G. Communication protocol

Communication protocol TCP/IP (Transmission Control Protocol/Internet Protocol) is currently the most used in the local area networks and Internet between 2 programs or 2 machines (a client and a server). The TCP layer ensures that all data sent is received by the receiving machine. The protocol of data exchange between the client and the server via the sockets (interfaces of connection) networks TCP/IP is modeled figure 6.

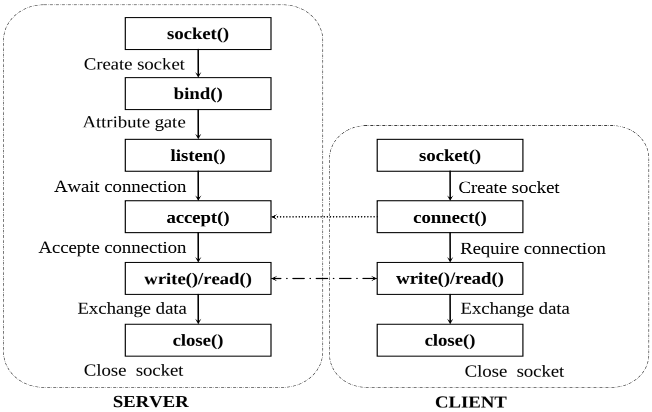


Fig. 6. Communication protocol TCP/IP

The wireless router is the central element of this experimentation; it is the only link between the whole of the robots, the module of vision and the module of intelligent control (see figure 5). It manages the connection of the robots to the management module (of the set of the sensors and the mobile sources) and to the parametric identification module. These various elements have each one a unique IP address thus allowing the dialogue between them. These elements open a socket of listening and expect a request for connection coming from another device. When a third machine tries to be connected to the host machine it activates connection and the dialogue between the two machines is set up. Data is exchanged in the form of data frames which are specifically coded.

H. Localization based on the vision

The robots Khepera III have wheels encoders able to calculate the relative displacement carried out at the time of the starting of the motors (odometric measurement). However, frictions between the wheels of the robot and the plate generate bias (in particular at the time of tight turns). In order to correct errors of trajectory, the robot location checking is implemented via image processing software (software SSL-vision [16-17]). This software uses the images provided by the camera and detects markers previously recorded (figure 5). These are used as a unique identifier; by placing a marker on each robot, it is possible to know its position (x_0, y_0) and its orientation (α_0) in a frame of a vector space Oxy. Once the new coordinates calculated, they are compared with the odometric measurements of the robots in order to estimate as well as possible the coordinates of the best robots on the plate. After measure treatments and process of quasi online identification (see paragraph III), each robot receives a frame of data which contains its current coordinates (x_0, y_0), its current orientation (α_0) and the desired position (x_1, y_1). Then, it calculates a trajectory to move from the current position (x_0, y_0, α_0) to the desired position (x_1, y_1), avoiding obstacles (if any). With the help of the SSL-vision software, the positions of the robots are updated every second to correct errors and avoid increasing the inaccuracy of the temperature measurements.

III. IDENTIFICATION

A. Direct problem

When all the parameters of the model are known $P = \{\Omega, T, \rho, c, \lambda, h, \phi_{heat}, \theta_0\}$, the system of partial derivatives equations in (1) is solved in order to obtain the temperature evolution $\theta(x, y; t)$ at any point of the plate and at every moment. This problem is called a well-posed problem because few noises on parameters P introduce small disturbances on state estimation $\theta(x, y; t)$. Once a parameter is unknown, an inverse problem is considered in order to proceed with its identification.

B. Inverse problem

In this communication, heat flux density is considered unknown and temperature measurement $\hat{\theta}_i(x, y; t)$ are realized. In order to estimate the unknown parameter, a method based on the minimization of the output error is implemented. The principle of this resolution is to find a value of unknown parameter $\phi_{heat}(x,y;t)$ that minimize the difference between predicted and real outputs

An estimator can be considered as:

$$\phi^*_{heat} = \operatorname{Arg\,min}\left(\frac{1}{2}\sum_{i=1}^{N}\int_{\mathfrak{T}}\left(\theta(C_i,t,\phi_{heat}) - \hat{\theta}_i(t)\right)^2 dt\right) \qquad (3)$$

Where N is number of sensors C_i and $\mathfrak{T}$ the time interval during which measures are taken into account.

In following paragraphs, sensor choice (fixed or mobile) strategies are considered and the definition of the intervals of observations $\mathfrak{T}$ is discussed.

C. The conjugate gradient method (Offline method)

To minimize the cost function defined in (3), the iterative Conjugate Gradient Method (CGM) is implemented [7]. An application to thermal engineering is presented in [8] and its regularization properties are illustrated in [9]. The selected algorithm is solving iteratively three well posed problems on a time interval.

- Direct problem (1) corresponding to the estimated heat flux density ϕ_{heat} at iteration k and computation of the criterion (3).
- Adjoint problem (4) written with Lagrangian formulation to compute the gradient of the cost function and the descent direction at iteration k.
- Sensitivity problem (5): calculation of the sensitivity function defined as the variation of temperature induced by variation of the heat flux density (in the descent direction).

At the end of the resolution of these three problems, a new value of ϕ^{k+1}_{heat} is calculated.

$$\begin{cases} \forall(x,y;t)\in\Omega\times\mathfrak{T} & \rho c\dfrac{\partial\psi}{\partial t} + \lambda\Delta\theta = E + \dfrac{2h\psi}{e} \\ \forall(x,y)\in\Omega & \theta(x,y;\tau) = 0 \\ \forall(x,y;t)\in\partial\Omega\times\mathfrak{T} & -\lambda\dfrac{\partial\psi(x,y;t)}{\partial\vec{n}} = 0 \end{cases} \qquad (4)$$

For the Adjoint problem (4), $E(x,y;t)$ depends on the difference between the estimated temperature and measurements at sensor position. Note that this problem is retrograde in time ($\tau = t_f$ and t_f is equal to the end of the time interval).

$$\begin{cases} \forall(x,y;t)\in\Omega\times\mathfrak{T} & \rho c\dfrac{\partial(\delta\theta)}{\partial t} - \lambda\Delta(\delta\theta) = \delta\Phi^k \\ \forall(x,y)\in\Omega & (\delta\theta)(x,y;0) = 0 \\ \forall(x,y;t)\in\partial\Omega\times\mathfrak{T} & -\lambda\dfrac{\partial(\delta\theta)(x,y;t)}{\partial\vec{n}} = 0 \end{cases} \qquad (5)$$

The variation of the heat flux density used in sensitivity problem is:

$$\delta\Phi^k = \frac{\left(\delta\phi^k_{heat}\right) - 2h(\delta\theta)}{e}$$

Problems (4) and (5), the equations to obtain the gradient (with $\psi(x,y;t)$) and the formulation of the descent depth are explained in [5, 12-14]. The iterative process continues until the cost function (3) is under a permissible limit (6) [2]:

$$J_{stop} = \frac{1}{2}n\sigma^2 \qquad (6)$$

Where n is number of measurements C_i considering on time interval $\mathfrak{T}$ and σ the standard deviation of the noise measurements (assumed as a Gaussian profile with mean value equal to zero).

This method is offline when $\mathfrak{T} = T$, the identification process begins only when all measurement are collected. It is thus necessary to wait the end of the experiment to identify the trajectory and power of heating sources.

D. Adaptation for a quasi-online method

For identification during the process, when measurements are available, the previous method should be adapted. It's possible to assume a set of time interval $\mathfrak{T}_j \subset T$ as $\bigcup\mathfrak{T}_j = T$. The choice of these sliding time intervals (start, length, recovery rates), is explained in [14] where several strategies are presented. Note in [14] that the experimental design and the choices of sensor procedures are not presented. Techniques for online identification are based on the behavior of the minimization algorithm to handle compromises between quality of the estimate and delay between estimation and measurements.

The algorithm is started only if new observations are not in adequacy with predicted temperature evolution. Lengths of time intervals must be choice considering the dynamics of the experimentation (priori experimental knowledge can be taken into account). Knowledge on the physical system studied is an undeniable asset to 'adjust' the online procedure. This note also applies to the choice of sensors presented in the next paragraph.

In the considered situation, it is shown that offline identification leads to results 70 minutes after the end of the experiment of 10 minutes, the online method provides satisfactory results only 1 minute after the end. This approach also allows estimation of the unknown parameter even before the end of the process (the average difference between the measurements and the results of online method is about 30 seconds).

E. Choice of relevant sensors.

In order to identify unknown trajectories of heating sources, a network of fixed sensors can be used. Indeed, temperature sensors (thermocouples) are not very expensive. However, this type of sensors introduces a non-negligible noise on measurement. On a plate (9m²), it's clear that several sensors are not sensitive to sources displacement. Also the problem concerns the selection of a few "relevant" sensors within a large number of "blind" sensors.

In [19], methods based on the optimal design of experiments are presented. In general, these approaches require a priori knowledge of a nominal value for the unknown parameter. Below, the proposed method is based on analysis of the iteratively solved sensitivities functions (5).

On the time interval $\mathcal{T}$ the sensors with sufficient sensitivity are selected (an example inspired by this approach is presented in [20]). One possible strategy would be, for an example, if you are looking 4 relevant sensors, to divide the interval $\mathcal{T}$ into 4 subintervals $\mathcal{T}^i$ of equal length, and on each, selected the sensor with the best sensitivity (norm sense) $L^2\left(\mathcal{T}^i\right)=\int_{\mathcal{T}^i}\left(\delta\theta\left(x,y;t\right)\right)^2 dt$. This would have enough signal on the data collected during $\mathcal{T}$. The previous method can be performed offline $\left(\mathcal{T}=T\right)$ or on sliding time intervals $\left(\mathcal{T}_j\subset T\right)$. An illustration of this method is proposed below for identification the heat flux density of a mobile heating source moving in thin plaque (1m²) with an offline CGM. Noisy measurements are available from 16 fixed sensors. In order to reduce the influence of noise measurement on the cost function J_{stop} (6), 4 least relevant sensors are rejected during the iterative process based on norm $L^2\left(\mathcal{T}^i\right)$. In Fig 7, is presented, 12 sensors (at different iterations) used to estimate the unknown heat flux density.

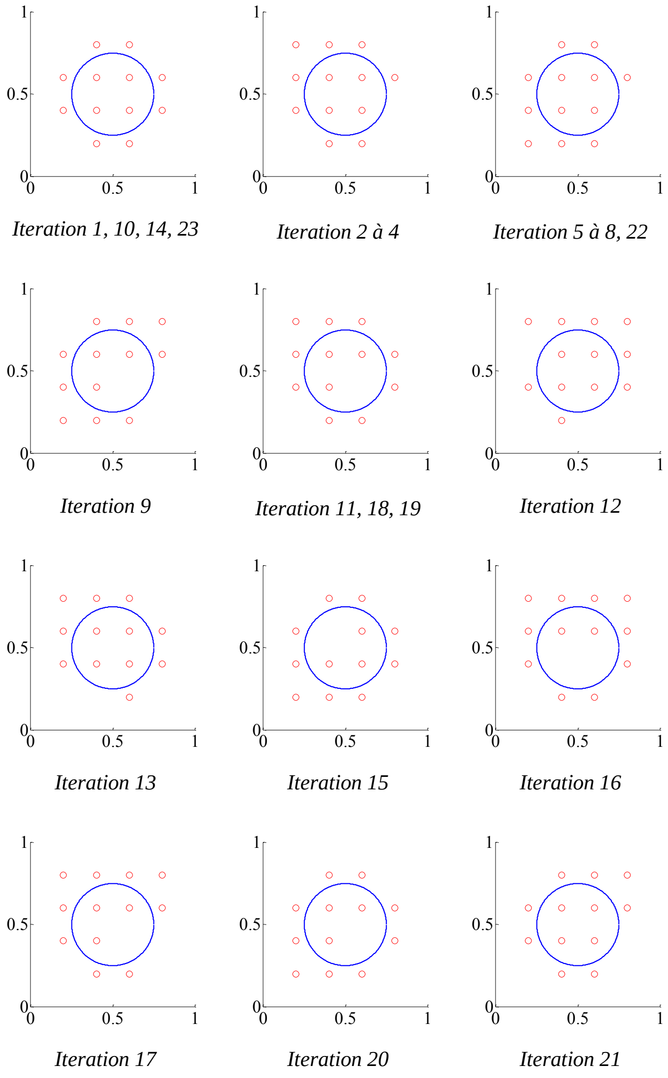


Fig. 7: Choice of sensors with offline method.

The choice of the most relevant sensors during the iterative process of minimization is not based on priori information. It is based on the results of the identification and in particular the analysis of the functions of sensitivity (in the direction of descent $\delta\Phi^k$); see problem (5).

At each iteration of the minimization algorithm, it is easy to estimate the relevant spatial areas in time and select sensors. The initialization of the 12 sensors amongst 16 (see iteration 1) is made to cover the plate without a priori.

In figure 7, according to the iterations and directions preferred by the CGM algorithm, several less relevant sensors are abandoned. It is particularly interesting that this method requires no a priori information unlike sensors selection techniques [19] that are based on an expected nominal location near unknown configuration. The estimation of the heat flux density are presented in Fig. 8.

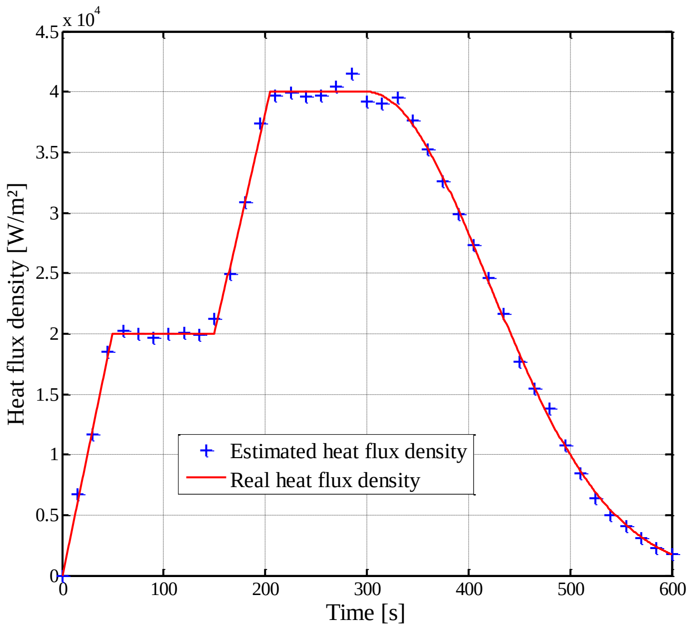


Fig. 8. Heat flux density identification at iteration 23.

The results of the identification process are obtained with 12 sensors and presented figure 8 are particularly satisfactory. In 23 iterations, the average residue between calculated and measured temperature is equal to $-5.10^{-3}\,\mathrm{K}$ and standard deviation of residues is equal to 0.48 K (which is of the order of magnitude of noise). The duration of identification on a standard laptop is about 40 minutes.

This concept can be used in strategies of mobile sensors deployment.

III. Conclusion and outlooks

A complete set (experimentation, definition of the inverse problem, resolution offline and online) is presented in this communication to examine different strategies for mobile sources tracking observation. Where the State is described by a system of parabolic PDE (thermal), the selection of the relevant sensors within a fixed network or the definition online of the trajectories of mobile sensors is discussed. The prospects envisaged at the end of this work consist in the realization of many experimental campaigns and to the study of non-linear systems for which input parameters would depend on the State. This work was partly funded with ANR-12-BS03-008-03.

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